ILLINOIS JOURNAL OF MATHEMATICS
Volume 34, Number 4, Winter 1990

TRANSFER OF INFORMATION ABOUT βN − N VIA OPEN REMAINDER MAPS[1]

BY

ERIC K. VAN DOUWEN

0. Conventions

All spaces are completely regular, and Hausdorff of course. We use X^* to denote $\beta X - X$, and $\mathbf{N}$, $\mathbf{Q}$ and $\mathbf{R}$ to denote the nonnegative integers, the rationals and the reals.

A *map* is a continuous function. The *Stone extension* of a map f: $X \to Y$ is the function $\beta X \to \beta Y$ which extends f; it will be denoted by βf. We use f^*, the *remainder map*, to denote the restriction $\beta f \restriction X^*$. Recall from [G1] that f^* maps X^* into Y^* if (and only if) f is perfect ($\equiv$ closed + compact fibers); hence f^* maps X^* onto Y^* if f is a perfect map from X onto Y.

The closure operators in X, βX and X^* are denoted by cl, Cl and Cl*. We use a similar convention for the interior operators int, Int and Int*.

We remind the reader that a space X is *realcompact* if for each $x \in X^*$ there is a G_δ-subset G of βX with $x \in G \subseteq X^*$. (This is equivalent to the original definition). Clearly Lindelöf spaces are realcompact.

1. Introduction

$\mathbf{N}^*$ is one of the most intensely studied spaces; so it is worthwhile to have tools available to transfer information about $\mathbf{N}^*$ to information about other Čech-Stone remainders. Two such tools, which are available already, are:

T_1. *C-embedded copies of* $\mathbf{N}$. Assume $\mathbf{N}$ can be embedded in X as a C-embedded subspace (this happens iff X is nonpseudocompact). Then $\mathbf{N}$ is closed in X, and $\mathrm{Cl}\,\mathbf{N} = \beta\mathbf{N}$, so $X^* \cap \mathrm{Cl}\,\mathbf{N} = \mathbf{N}^*$. The fact that $\mathbf{N}$ is C-embedded in X gives information about the way $\mathbf{N}^*$ fits inside X^*; cf. [R, 4.5(d)], [I], [GJ, 9M], [F]. (An example in [vD_2, 3] shows that it is not sufficient to know that $\mathbf{N}$ is closed and C^*-embedded in X.) Rudin's proof of the implication (a) $\Rightarrow$ (b) in Theorem 4.1 is an early example.

Received November 6, 1978.

[1]This paper was originally accepted subject to revision in 1978. The revised manuscript was found in the author's files after his death on July 28, 1987. His obituary appeared in Topology and its Applications, vol. 31, number 1.