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MANIFOLDS WITH INFINITELY MANY ACTIONS OF AN ARITHMETIC GROUP[1]

BY

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It is well known that if Γ is a lattice in a simple Lie group of higher split rank then in any finite dimension Γ has only finitely many inequivalent linear representations. This is one manifestation of the strong linear rigidity properties that such groups satisfy. When one considers non-linear representations, say smooth actions of Γ on compact manifolds, one still sees a large number of rigidity phenomena [7]. This is particularly true for actions preserving a connection. On the other hand, the point of this note is to establish the following result.

THEOREM 1. *Let G be the Lie group $SL(n, \mathbf{R})$, $n \geq 3$, or $SU(p, q)$, $p, q \geq 2$. Then there is a cocompact discrete subgroup $\Gamma \subset G$ and a smooth compact manifold M such that there are infinitely many actions of Γ on M with the following properties*:

i) *The actions are mutually non-conjugate in* Diff(M), Homeo(M), *and* Meas(M), *where the latter is the group of measure class preserving automorphisms of M as a measure space*;

(ii) *Each action leaves a smooth metric on M invariant, is minimal* (*i.e., every orbit is dense*), *and ergodic* (*with respect to the smooth measure class.*)

Theorem 1 is easily deduced from a certain non-rigidity phenomenon for tori in compact semisimple groups. Namely, fix a compact semisimple Lie group C and call closed subgroups H_1 and H_2 equivalent if there is an automorphism α of C such that $\alpha(H_1) = H_2$. We can then ask to what extent the diffeomorphism class of C/H determines the equivalence class of H. (The natural question is under what circumstance the map from equivalence classes of (a class of) closed subgroups to diffeomorphism classes of manifolds is finite-to-one.) Here we show:

THEOREM 2. *Let $C = SU(n) \times SU(n)$, $n \geq 2$. Then there is a family of mutually non-equivalent tori T_k, $k \in \mathbf{Z}^+$, such that C/T_k are all diffeomorphic.*

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