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# HOMOGENIZABLE RELATIONAL STRUCTURES

BY

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In this paper we consider certain classes of relational structures which do not have the Amalgamation Property, and hence do not have a universal homogeneous structure, but which can be made to satisfy the Amalgamation Property by the imposition of some additional structure. We thus obtain a class which is the age of a homogeneous structure, and which has the original class as 'underlying structures.' We show that the underlying structure of the countable universal homogeneous structure has a model complete theory, so is a model comparison for its universal theory.

In Section 1 we first define homogenizations and homogenizable classes then introduce the notion of Local Failure of Amalgamation. Section 2 is devoted to stating and proving Theorem 2.1, which gives a sufficient condition for a class of structures to be homogenizable. This condition involves Local Failure of Amalgamation. In Section 3 we show that the theory of the 'homogenized' structure is model complete.

## 1. Homogenizable structures

DEFINITION 1.1. Let $\mathscr{L}$ be a purely relational first-order language, and let $\mathscr{C}$ be a class of $\mathscr{L}$-structures. The age of an $\mathscr{L}$-structure $\Gamma$ is the class of structures isomorphic to finite substructures of $\Gamma$. $\Gamma$ is homogeneous if every isomorphism between finite substructures of $\Gamma$ extends to an automorphism of $\Gamma$. $\mathscr{C}$ has the *Hereditary Property* (HP) if all substructures of members of $\mathscr{C}$ belong to $\mathscr{C}$. $\mathscr{C}$ has the *Joint Embedding Property* (JEP) if whenever $A, B \in \mathscr{C}$, there is a structure $D \in \mathscr{C}$ embedding both $A$ and $B$. $\mathscr{C}$ has the *Amalgamation Property* (AP) if for all embeddings $\alpha\colon C \to A$ and $\beta\colon C \to B$ between C-structures $A$, $B$ and $C$, there is a structure $D \in \mathscr{C}$ and embeddings $\gamma\colon A \to D$ and $\delta\colon B \to D$ such that $\alpha\gamma = \beta\delta$.

In 1953, Fraïssé showed that the Amalgamation Property is a crucial condition for the existence of a homogeneous structure with a given age.

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