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DISTANCE SPHERES AND MYERS-TYPE THEOREMS FOR MANIFOLDS WITH LOWER BOUNDS ON THE RICCI CURVATURE

BY

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Introduction

Let M be a complete connected riemannian manifold of class C^r, $r \geq 3$, and dimension $d \geq 2$. One of the results still viewed by many to be one of the most important as well as the loveliest concerning the global properties of such a space is the following work of S. Myers (1941).

THEOREM. *Suppose that the Ricci curvature of M is bounded from below by a positive constant m. Then the diameter of M is no larger than*

$$\pi\sqrt{(d-1)/m}\,.$$

In particular, M is compact.

Here, the Ricci curvature is viewed as a function on the unit tangent bundle of M. Attempts at generalizing and refining this theorem have received considerable attention. Most notably, there are the works of W. Ambrose [1], E. Calabi [4, 5], A. Avez [2], S.T. Yau [18, 19], K. Shiohama [17], G.J. Galloway [9], S. Markvorsen [13], and J. Cheeger, M. Gromov, and M. Taylor [7]. In the present paper, our purpose is to prove

MAIN RESULTS (Theorems 3.3 and 3.5). *Let m be any given constant, not necessarily positive. Assume that the Ricci curvature of M is bounded below by (resp. strictly greater than) m. Suppose that there exists a point $p \in M$ and a number $r \in \mathbb{R}_+$ such that the distance sphere in M with center p and radius r has mean curvature away from its singularities greater than (resp. greater than or equal to $\sqrt{|m|}$. Then the diameter of M has a finite upper bound, and hence M is compact. In the first case, the upper bound on the diameter can be explicitly estimated in terms of the supremum of the mean curvature.*

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