ILLINOIS JOURNAL OF MATHEMATICS
Volume 35, Number 1, Spring 1991

THE CLASS OF SYNTHESIZABLE PSEUDOMEASURES

BY

A. S. KECHRIS,[1] A. LOUVEAU AND V. TARDIVEL

In this paper we study descriptive set theoretic questions related to concepts of harmonic synthesis on the unit circle $\mathbf{T}$, and their relationship with the structure of uniqueness sets.

We denote by $A = A(\mathbf{T})$ the space of functions on $\mathbf{T}$ with absolutely convergent Fourier series, by PM the space of pseudomeasures on $\mathbf{T}$ and by PF the space of pseudo-functions on $\mathbf{T}$. Thus $PF^* = A$, $A^* = PM$. Finally $K(\mathbf{T})$ denotes the compact space of closed subsets of $\mathbf{T}$ with the Hausdorff metric. The three basic notions associated with harmonic synthesis are the following:

(i) A function $f \in A$ *satisfies synthesis* if $\langle f, S \rangle = 0$ for all $S \in PM$ with $f = 0$ on $\operatorname{supp}(S)$.

(ii) A pseudomeasure $S \in PM$ *satisfies synthesis* if $\langle f, S \rangle = 0$ for all $f \in A$ with $f = 0$ on $\operatorname{supp}(S)$. This is equivalent to saying that $S \in N(\operatorname{supp}(S))$, where for each $E \in K(\mathbf{T})$, we let

$M(E)$ = space of (Borel complex) measures whose (closed) support is contained in E,

$N(E)$ = weak*-closure of $M(E)$.

For simplicity, if $S \in PM$ satisfies synthesis, we will call it a *synthesizable* pseudomeasure.

(iii) A set $E \in K(\mathbf{T})$ is a *set of synthesis* if for all $f \in A, S \in PM$ with $\operatorname{supp}(S) \subseteq E$ and $f = 0$ on E we have $\langle f, S \rangle = 0$. Equivalently, if

$$I(E) = \{f \in A : f = 0 \text{ on } E\},$$
$$J(E) = \{f \in A : f = 0 \text{ on an (open) nbhd of } E\},$$

E is of synthesis iff the strong closure of $J(E)$ in A is equal to $I(E)$. Also equivalently, E is of synthesis iff $N(E) = PM(E)$ (= the space of pseudomeasures supported by E).

Received November 22, 1988.
[1]Research partially supported by the National Science Foundation.