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STRICT SINGULARITY OF INCLUSION OPERATORS BETWEEN γ-SPACES AND MEYER-KÖNIG ZELLER TYPE THEOREMS

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Introduction

Let E, F be BK-spaces containing Φ and having $F \subset E$. Then $F < E$ means that whenever G is a BK-space containing Φ and satisfying $E = F + G$, then $E = G$ holds. It is known for instance that $c_0 < l^\infty$, (cf. [10]), and that $l^p < l^q$ for $1 \leq p < q \leq \infty$ (cf. [9]). Also $l < E$ is known to be valid for every BK-space E into which l is weakly compactly included (cf. [9]). In a certain sense $F < E$ indicates that F is a small subspace of E.

In [11], Snyder has shown that $F < E$ is valid if and only if the inclusion operator $F \to E$ is strictly cosingular in the sense of Pelczynski (also called a Pelczynski operator). This clearly throws new light on the above examples and, moreover, provides various other situations $F < E$ (see [11]). Strict cosingularity of an operator has a dual description. In our situation it tells that the inclusion operator i: $F \to E$ is strictly cosingular if and only if its adjoint, the restriction operator i': $E' \to F'$ is strictly singular in the weak star sense. In [11], Snyder proved that the latter is equivalent to strict singularity of i' with respect to the dual norms in the case where F is a separable space.

From the point of view of sequence space theory it seems desirable to express the relation $F < E$ in terms of β- or γ-duality rather than abstract topological duality. In [9], Snyder has pursued this program, discussing a property of the inclusion operator $E^\gamma \to F^\gamma$, called the Meyer-König Zeller property (MKZ property for short), which in many cases gives rise to a dual version of $F < E$. Closing the circle in [11], Snyder introduced an abstract version of the MKZ property in Banach spaces and proved that the restriction operator i': $E' \to F'$ has this abstract MKZ property precisely when it is strictly singular.

The abstract dual description of $F < E$ being complete in the case where F is separable, this is far from being true in the case of concrete duality. The result in [9] expressing $F < E$ in terms of MKZ for $E^\gamma \to F^\gamma$ requires that both E, F are BK-AD-spaces and that, in addition, the closure E_0' of Φ in E'

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