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Z_2-GRADED ALGEBRAS

BY

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1. Introduction

This paper was inspired by [2]. Recast into the language of Lie superalgebras, rather than Lie algebra square roots, one of Mackey's results reads as follows. Let L be a Lie superalgebra with even part H and odd part N. Assume that N is two-dimensional, that H is three-dimensional, and that $N^2 = H$. Then either $HN = 0$ or L is the unique five-dimensional simple Lie superalgebra (the first of the orthosymplectic series). In Theorem 1 I exhibit a generalization. Any field of characteristic $\neq 2$ is admissible in Theorem 1.

I stubbornly sought to fit in characteristic 2 as well. But in characteristic 2 the notions of Lie algebra and Lie superalgebra coincide. So it was natural to make a parallel study for Lie algebras. The result was Theorem 2 in which, however, characteristic 3 was an unexpected exception.

The final section of the paper contains a number of additional remarks.

2. Lie superalgebras

Note that in all algebras the operation is being written simply as multiplication.

THEOREM 1. *Let $L = H + N$ be a Lie superalgebra with even part H and odd part N. Infinite-dimensionality is permitted and the base field can be any field of characteristic $\neq 2$. Assume that the multiplication $N \times N \to H$ is the symmetric tensor product, i.e., is as free as possible. (The mapping is not assumed to be onto.) Then there exists a unique alternate form $(\ ,\ )$ on N such that*

$$(1) \qquad xy.z = (y,z)x + (x,z)y$$

for all $x, y, z \in N$.

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