ILLINOIS JOURNAL OF MATHEMATICS
Volume 35, Number 1, Spring 1991

CHARACTERIZATIONS WITHOUT CHARACTERS

BY

RON SOLOMON[1] AND S.K. WONG

1. Introduction

The current program of revision of the classification of the finite simple groups has led us to approach old characterization problems via new avenues. The direction of the new approaches has made it natural to look for new ways to complete the characterizations, more consistent with the new avenues of approach.

For example, the group A_7 was characterized by Michio Suzuki in 1959 in a paper [10] which made extensive and detailed use of character theory. This characterization was invoked by Gorenstein and Walter to complete their "Dihedral Paper" [9] and, again, by Bender in his revision of the Gorenstein-Walter theorem [5]. However Bender invokes Suzuki at a point where both the centralizer of an involution and the group order are known. At this point a far more elementary and character-free argument is available and this is provided in Sections 3 and 5 of this paper. In thinking about this we realized that combining some of Bender's arguments with our own affords an almost character-free proof of the following case of the Dihedral Theorem.

THEOREM 1.1. *Let G be a finite simple group with a dihedral Sylow 2-subgroup. Suppose that the centralizer H of an involution of G is* **properly** *contained in a subgroup $\tilde{H}$ of G with $F^*(\tilde{H}) = F(\tilde{H})$. Then G is isomorphic to either A_5, $PSL(3,2)$, $PSL(2,9)$ or A_7.*

The lion's share of the proof of (1.1) is in Bender [5]. After some preliminaries in Sections 2, 3, 4 and 5, we outline Bender's reductions (with some improvements) and the completion of the proof of (1.1) in Section 6.

The remainder of the paper is devoted to a character-free proof of Brauer's well-known result [7]:

THEOREM 1.2. *Let G be a finite simple group with an involution $t \in G$ such that $H = C_G(t) \cong GL(2,3)$. Then G is isomorphic either to M_{11} or to $PSL(3,3)$.*

Received October 19, 1988.

[1]Partially supported by a grant from the National Science Foundation.