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OUTER AUTOMORPHISMS OF GROUPS

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1. Introduction

In this paper we concern ourselves with the group of outer automorphisms of groups. It is obvious that if G is a group embedded in another group H, then any inner automorphism of G lifts to an (inner) automorphism of H. Angus Macintyre asked whether this extension property actually characterizes inner automorphisms. The answer is affirmative as recently shown by Schupp [24]. He actually proved the following result which is stronger than the one stated in [24].

THEOREM [24]. *If G is a group then there exists an extension H of G with* $\operatorname{Out} H = 1$ *(i.e., H is complete) and* $\mathfrak{n}_H(G) = G$.

$\operatorname{Out} G$ denotes the factor group $\operatorname{Aut} G/\operatorname{Inn} G$ where $\operatorname{Inn} G$ is the normal subgroup of all inner automorphisms of G.

This theorem generalizes an earlier countable version due to Miller, Schupp [17]. We want to put Macintyre's question into a more general setting which will lead quite naturally to new questions. For instance, is every group the outer automorphism group $\operatorname{Out} G$ of some group G? A positive answer to the latter will be part of our main theorem. This is in contrast with results due to Robinson [22], [23] who showed that *not* every (finite) group is the automorphism group of some group. For instance, A_n, $n \neq 2, 8$, is not isomorphic to an automorphism group. In Section 3 we will provide a completely different proof of Schupp's theorem, which has the following three advantages. First of all, it gives an answer to the extension of Macintyre's question. Secondly, we use only very elementary group theory without "elements of order 160 or 81". The small cancellation theory used in [24] clearly has developed into a beautiful theory over the last 30 years [16], [25], however, some of its combinatorial details require technical computations. Therefore it might be desirable to have a "pure" group theoretic proof. Finally, our construction is close to abelian groups. This allows our extension

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