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L^p AND SOBOLEV SPACE MAPPING PROPERTIES OF THE SZEGÖ OPERATOR FOR THE POLYDISC

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1. Introduction

Suppose Ω is a domain and $\partial\Omega$ is its boundary. The Szegö operator $\mathscr{S}$ for $\partial\Omega$ is defined to be the orthogonal projection of $L^2(\partial\Omega)$ into $H^2(\partial\Omega)$ where $H^2(\partial\Omega)$ consists of those functions in $L^2(\partial\Omega)$ which are the extensions of holomorphic functions in Ω. It is well known (see [2], p. 55) that the Szegö operator may be expressed as an integral operator of the form

$$\mathscr{S}f(z) = \int_{\partial\Omega} \tilde{S}(z,\zeta) f(\zeta)\, d\sigma(\zeta)$$

where $\tilde{S}$ is the Szegö kernel.

Recently it has been shown (see [1]) that the Szegö operator for the topological boundary of the bidisc in $\mathbf{C}^2$ with respect to Lebesgue surface area measure is bounded on L^p and L^p_α for $1 < p < \infty$ and $\alpha > 0$. In this paper we show that the same results hold for the topological boundary of the polydisc in $\mathbf{C}^n$ for $n \geq 3$. Furthermore one may have arbitrary radii for the polydisc in each dimension and obtain the same results for any n.

The proofs of these results use the Marcinkiewicz Multiplier Theorem in order to reduce the problem to considering a more tractable operator than the Szegö operator. It turns out that the "tractable" operator is simply the composition of $n - 2$ Bergman operators for the disc in $\mathbf{C}$ and of the Szegö operator for the topological boundary of the bidisc in $\mathbf{C}^2$.

We point out to the reader that the mapping properties for the Szegö operator for the distinguished boundary of the polydisc are trivial and should not be confused with the subject of this paper.

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