ILLINOIS JOURNAL OF MATHEMATICS
Volume 35, Number 2, Summer 1991

REPRESENTING MEASURES ON MULTIPLY CONNECTED PLANAR DOMAINS

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The linear functional $f \to f(a)$ of evaluation of an analytic function f at a point a in a g holed bounded planar domain admits representation in the form $f(a) = \int_{\partial D} f\,dm$, where the non-negative measure m supported on the boundary ∂D of D belongs to the g dimensional compact convex set M_a of representing measures for a. This convex set M_a of representing measures is a subset of the vector space $M_{\mathbf{R}}(\partial D)$ of real Borel measures on ∂D. By fixing a natural basis, the convex set M_a can be affinely identified with a convex set C_a in $\mathbf{R}^g$. Throughout this paper it will be assumed that the positively oriented boundary of D is the union

$$\partial D = b_0 \cup b_1 \cup \cdots \cup b_g$$

of the disjoint simple closed analytic curves $b_0, b_1, \ldots, b_g$ with $b_1, \ldots, b_g$ the boundaries of the holes and b_0 the boundary of the unbounded component of the complement.

It will be shown that the convex set C_a has the smooth parametrization π_a: $\mathbf{R}^g \to C_a$ given by

$$\pi_a(x) = \frac{1}{2\pi}\vec{\nabla}\left\{\log \frac{\theta(x)}{\theta(x + \omega_a)}\right\}, \tag{0.1}$$

where θ is the Riemann theta function associated with the Schottky double X of D. The vector constant ω_a appearing in (0.1) is $\omega_a = (\omega_1(a), \ldots, \omega_g(a))$, where $\omega_j(a)$ is the harmonic measure of $b_j (j = 1, \ldots, g)$ based at a. Since the θ function is $\mathbf{Z}^g$ periodic, then π_a provides a covering of C_a by the real g dimensional torus $\mathbf{T}_0 = \mathbf{R}^g/\mathbf{Z}^g$.

The parametrization (0.1) can be explained in the following manner. Let

$$\operatorname{Jac}(X) = \mathbf{C}^g/(\mathbf{Z}^g + \tau\mathbf{Z}^g)$$

Received March 17, 1989.

1980 Mathematics Subject Classification (1985 Revision). Primary 46J10; Secondary 46E20, 46J25, 47A67.