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COMPLETION REGULAR MEASURES ON PRODUCT SPACES WITH APPLICATION TO THE EXISTENCE OF BAIRE STRONG LIFTINGS[1]

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1. Introduction

In [5] and [13] it is proved that, subject to the continuum hypothesis (CH), every (positive) Radon measure supported on a space with a topology basis of card $\leq c$ admits a Baire lifting as well as a Borel strong lifting.

On the other hand, relatively little is known about the existence of Baire liftings that are, at the same time, strong. On the positive side, Baire strong liftings have been shown to exist, when CH is assumed, in a very restricted class of measures: on any product of less than or equal to $\aleph_2$ supported Radon measures, each on a compact metric space ([11]—see also [15]). We also mention that D. Maharam in [12] proved that existence of a (completion) Baire strong lifting for the product measure, if each compact space is either a closed unit interval or two point space, without assuming CH and for any number of factors. On the negative side, D.H. Fremlin has exhibited a completion regular measure on $[0,1]^{\aleph_2}$ that admits no strong lifting [6].

The question that naturally arises is: what (completion regular) measures on $[0,1]^{\aleph_1}$ admit a Baire strong lifting?

We prove that, under CH, the answer is always positive. The proof, in the spirit of [11], is based on a special characterization of a class of "maximal" open sets, for a completion regular measure on an arbitrary product of compact spaces (Lemma 2).

In the sequel, trying to find Baire strong liftings for a product measure, we investigate -in connection with some questions posed in [3] and [2]-which products of two compact completion regular measure spaces (X, μ), (Y, ν) are completion regular. We establish that, subject to Martin's Axiom and the negation of the continuum hypothesis, such a product is completion regular, provided that one of these topological factors is of the form $\Pi_{j \in J} Y_j$, with the Y_j compact metric spaces (Theorem 2).

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[1]This work is related to some parts of the second author's dissertation, written in 1987.