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ADDITIVE DERIVATIONS OF SOME OPERATOR ALGEBRAS

BY

PETER ŠEMRL[1]

1. Introduction

All algebras and vector spaces in this note will be over $\mathbf{F}$ where $\mathbf{F}$ is either the real field or the complex field. Let $\mathscr{A}$ be an algebra and $\mathscr{A}_1$ any subalgebra of $\mathscr{A}$. An additive (linear) mapping $D\colon \mathscr{A}_1 \to \mathscr{A}$ is called an additive (linear) derivation if

$$(1) \qquad D(ab) = aD(b) + D(a)b$$

holds for all pairs $a, b \in \mathscr{A}_1$. Let X be a normed linear space. By $\mathscr{B}(X)$ we mean algebra of bounded linear operators on X. We denote by $\mathscr{F}(X)$ the subalgebra of bounded finite rank operators. We shall call a subalgebra $\mathscr{A}$ of $\mathscr{B}(X)$ standard provided $\mathscr{A}$ contains $\mathscr{F}(X)$.

This research is motivated by the well-known results in [2], [3].

THEOREM 1.1. *Let X be a normed space and let $\mathscr{A}$ be a standard operator algebra on X. Then every linear derivation $D\colon \mathscr{A} \to \mathscr{B}(X)$ is of the form*

$$D(A) = AT - TA$$

for some $T \in \mathscr{B}(X)$.

THEOREM 1.2. *Let $\mathscr{A}$ be a semi-simple Banach algebra. Let $D\colon \mathscr{A} \to \mathscr{A}$ be an additive derivation. Then $\mathscr{A}$ contains a central idempotent e such that $e\mathscr{A}$ and $(1-e)\mathscr{A}$ are closed under D, $D|_{(1-e)\mathscr{A}}$ is continuous and $e\mathscr{A}$ is finite dimensional.*

Using these two results one can easily see that every additive derivation $D\colon \mathscr{B}(X) \to \mathscr{B}(X)$, where X is an infinite dimensional Banach space, is inner. In this note we shall give a complete description of all additive derivations on

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