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SEVERAL RESULTS CONCERNING UNCONDITIONALITY IN VECTOR VALUED L^p AND $H^1(\mathscr{F}_n)$ SPACES

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1. Introduction

Recently, vector valued versions of several results concerning basis properties of L^p spaces have been obtained for the spaces $L^p(E)$ where E is a UMD space. In particular, T. Figiel [Fi] has shown that the Haar and Franklin systems are equivalent in $L^p(E)$, $1 < p < \infty$. The main technical result of the present paper, Theorem 2 below, is of a similar nature; one shows that certain "Haar-like" sequences in $L^p(E)$, $1 < p < \infty$, are equivalent to sequences spanning all of an $L^p((\Omega, \mathscr{F}, p), E)$ space. The operator used for this equivalence is closely related to the one used by Maurey in [Ma1] and [Ma2]. An argument of Herz, also used by Maurey, is then used (Theorem 4) to show that a similar equivalence holds in $H^1(\mathscr{F}_n, E)$ spaces (see notations below for the definition of these spaces).

As corollaries, one gets vector valued versions of the Gamlen-Gaudet theorem, characterizing the isomorphic structure of subsequences of the classical Haar functions in L^p. These versions extend also to the finite dimensional case as well as for the H^1 case. The approach here follows the first author's paper [Mü1]. These results are contained in Theorem 3 and Corollary 8.

Another corollary to Theorems 2 and 4 (Corollary 7) is that, if E is UMD then $H^1(\mathscr{F}_n, E)$ has an unconditional decomposition into copies of E and if E has in addition an unconditional basis, then so does $H^1(\mathscr{F}_n, E)$. This extends a result of Maurey stated in [Ma1].

2. The main technical result

Let $(\Omega, \mathscr{F}, |\cdot|)$ be a given probability space. Let E be a Banach space. Then we denote by $L^p(\Omega, \mathscr{F}, |\cdot|, E)$ (or simply by $L^p(E)$) the Banach

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