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ON THE BRAUER GROUP AND QUOTIENT SINGULARITIES

BY

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Let k be an algebraically closed field with characteristic 0. Let $\mathrm{B}(\cdot)$ denote the Brauer group functor as defined in [AG]. Let A be a regular local ring which is a k-algebra essentially of finite type. Let G be a finite group of k-automorphisms of A. Suppose no height 1 prime of A ramifies over A^G. Let P be a prime ideal of height ≥ 2 in A^G and let R be the local ring $(A^G)_P$. Set $S = A \otimes_{A^G} R$. Then G acts on S and $S^G = R$. So S is a finite R-module and no height 1 prime of S ramifies over R. If $K = K(A^G)$ denotes the quotient field, we have the following inclusion relations:

$$\begin{array}{ccccc} A & \subseteq & S & \subseteq & K(A) \\ \uparrow & & \uparrow & & \uparrow \\ A^G & \subseteq & R & \subseteq & K \end{array}$$

Therefore S is a localization of A in the field of fractions $K(A)$ hence is a regular domain. Since S is finite over R and R is local, S is a semilocal ring. We say that *the ring R has quotient singularities* if S is a local ring. The maximal ideals of S correspond to the prime ideals of A lying over P, so we see that R has quotient singularities if and only if there is a unique prime ideal Q of A lying over P.

In this short note, we investigate the kernel $\mathrm{B}(K/R)$ of the natural map $\tau\colon \mathrm{B}(R) \to \mathrm{B}(K)$. If R is regular, it is known that $\mathrm{B}(K/R) = (0)$ [AG, Theorem 7.2, p. 388]. For this reason we are primarily interested in the situation where R actually has singularities. This study was motivated by similar questions about the Brauer group and rational singularities on surfaces that were answered in Section 1 of [FS]. Theorem 1 below can also be considered an attempt to correct Theorem 12 of [DF] which is false; a counterexample is given in [DFM]. The example is a normal algebraic surface X with isolated rational singular point P such that $\ker \tau$ is finite and non-trivial.

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