FACTORIZATION OF SOLUTIONS OF CONVOLUTION EQUATIONS II

BY

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0. Let $C^{\infty}(\mathbb{R}^n)$, be the vector space of complex valued C^{∞} functions on \mathbb{R}^n . It is well known (from the fundamental Principle of Ehrenpreis for the case n > 1 and more easily, from the classical Euler exponential polynomial representation of solutions of ordinary differential equations, for the case n = 1) that if the partial differential equation

$$(0.1) Q(D)f = 0$$

is such that $Q \in \mathbb{C}[z_1, \ldots, z_n]$ can be factored as $Q = Q_1 \cdot Q_2$, with Q_1 and Q_2 relatively prime, then every C^{∞} solution of (0.1) can be written as $f = f_1 + f_2$, with $Q_i(D)f_i = 0$, i = 1, 2.

The natural extension of the previous result to convolution equations in the space $H(\mathbf{C})$ of entire functions was obtained by V.V. Napalkov [9]. This last result has been successively extended by the authors [6], for a class of spaces of which both $H(\mathbf{C})$ and $C^{\infty}(\mathbf{R})$ are particular cases, under natural hypotheses on the convolutors, without mentioning the Fundamental Principle, but employing specific properties of suitable spaces of entire functions satisfying certain growth conditions.

Recent results (Berenstein-Struppa [1], Meril-Struppa [7], Morzhakov [8]) obtained for convolution operators acting on the space $H(\Omega)$ of holomorphic functions on a convex domain Ω of C, now allow us to extend the result of [6] to this space.

In Section 1, we give the basic definitions for the rest of the paper, while in Section 2 we adapt the well known Hörmander's L^2 -theory to the spaces of Fourier-Borel transform of analytic functionals with prescribed carrier; a corona-like theorem is obtained (Theorem 2.1). The desired factorization results are finally obtained in Section 3.

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