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SOME REMARKS ON COMPLEX POWERS OF $(-\Delta)$ AND UMD SPACES

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Introduction and notations

If X is a Banach space, $(\Omega, \mathscr{A}, \mu)$ a measure space and $1 \leq p < +\infty$, we will denote by $L_p(\Omega, X)$ ($L_p(\Omega)$ if $X = \mathbf{R}$), the Banach space of classes of Bochner measurable functions f from Ω to X such that

$$\int_\Omega \|f(t)\|_X^p \, d\mu(t) < +\infty,$$

equipped with the norm

$$\|f\|_p = \int_\Omega \|f(t)\|_X^p \, d\mu(t)^{1/p}.$$

We will also denote by $C_0^\infty(\mathbf{R}, X)$ ($C_0^\infty(\mathbf{R})$ if $X = \mathbf{R}$) the space of C^∞-functions from $\mathbf{R}$ to X such that $\lim_{t \to \pm\infty} \|f(t)\| = 0$, equipped with the norm

$$\|f\|_\infty = \sup\{\|f(t)\|_X, t \in \mathbf{R}\}.$$

We recall that X is UMD if martingale differences with values in X converge unconditionally in $L_2(\Omega, X)$ where Ω is any probability space, that is: there exists a constant $C > 0$, such that whenever $(M_k)_{k \in \mathbf{N}}$ is a bounded martingale in $L_2(\Omega, X)$ and $(\varepsilon_k)_{k \in \mathbf{N}}$ is a choice of signs,

$$\left\| \sum_{k=1}^\infty \varepsilon_k d_k \right\|_2 \leq C \left\| \sum_{k=1}^\infty d_k \right\|_2 \quad \text{where } d_{k+1} = M_{k+1} - M_k.$$

By a martingale, we mean that there exists an increasing sequence of σ-subalgebras $(\mathscr{A}_k)_{k \in \mathbf{N}}$ of $\mathscr{A}$ such that $E^{\mathscr{A}_k}[M_{k+1}] = M_k$, where $E^{\mathscr{A}_k}$ is the conditional expectation with respect to $\mathscr{A}_k$. It is well known that this

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