ILLINOIS JOURNAL OF MATHEMATICS
Volume 35, Number 3, Fall 1991

RATIONAL PERIOD FUNCTIONS AND INDEFINITE BINARY QUADRATIC FORMS, II

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1. Introduction

The classification of rational period functions of positive weight was begun in [CP]. This was accomplished by looking at possible minimal or irreducible systems of poles. Knopp in [Kn2] showed that the finite poles of rational period functions occur only at zero or at real quadratic irrationals and then found all rational period functions with poles only at zero. Of interest now is the case where the minimal pole set contains quadratic irrationals. If the minimal pole set exhibits algebraic symmetry, that is, if the pole set contains α', the conjugate of α, whenever it contains α, then the rational period function of weight $2k$, k odd, with poles only at elements of the pole set is of the form [CP]

$$\sum_{\substack{a>0>c \\ [a,b,c]\in\mathscr{A}}} \frac{1}{(az^2 - bz + c)^k}$$

where the sum is over primitive indefinite quadratic forms in a narrow equivalence class $\mathscr{A}$.

The situation when the irreducible pole set fails to be symmetric is studied in this paper. Given a pole set which is minimal and not symmetric, it is always possible to construct a rational period function of weight $2k$ with quadratic irrational poles only at elements of that pole set whenever $k = 1$, 2, 3, 4, 5 or 7. For all other weights certain "obstructions" may occur in the construction which act as barriers to the existence of rational period functions. In fact such obstructions are contained in a space isomorphic to $S_{2k} \oplus S_{2k}$ where S_{2k} is the space of cusp forms of weight $2k$. By way of example an infinite family of irreducible pole sets is given for which it is impossible to construct a rational period function of weight twelve with quadratic irrational poles in exactly one of the pole sets.

Received June 23, 1989.
1980 Mathematics Subject Classification (1985 Revision). Primary 11F11; Secondary 11F67.