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CLOSED-FORM SOLUTIONS OF SOME PARTIAL DIFFERENTIAL EQUATIONS VIA QUASI-SOLUTIONS I

BY

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Dedicated to the fond memory of Allen Shields

1. Introduction

For many PDE's, closed form (or *explicit*) solutions are so hard to come by that *any* examples are valuable in themselves. This paper expounds a new method that finds closed-form solutions for several non-linear PDE's, including the Klein-Gordon, eikonal and (non-parametric) minimal surface equations. In Part II, to be published separately, the method will be used to get some new results on separation of variables in some of the PDE's of mathematical physics.

In principle, the method applies to *any* PDE, but requires some luck or special ingenuity in practice. Symbolic computation on electronic computers has been a big help with the often lengthy and complicated computations.

Two established methods of obtaining closed form solutions are the symmetry method of Sophus Lie et al. (see [DRE]), and the method of inverse scattering, of Kruskal, Lax, et al. (See [ZAS]).

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