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ON THE SPIN BORDISM OF $B(E_8 \times E_8)$

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Let E_8 be the exceptional Lie group; let BE_8 be its universal classifying space. Bott and Samuelson (2) have shown that in dimensions less than 16, the only non-zero homotopy of E_8 is $\pi_3(E_8) = Z$, $\pi_{15}(E_8) = Z$. By the long exact homotopy sequence of the universal E_8-bundle, $\pi_4(BE_8) = Z$, $\pi_{16}(BE_8) = Z$, and $\pi_k(BE_8) = 0$ for all other $k \leq 16$. Let $K(Z, 4)$ be the Eilenberg-MacLane space whose only non-trivial homotopy group is infinite cyclic in dimension 4. By the Whitehead theorem (see, e.g., Serre (5)) the map $BE_8 \to K(Z, 4)$, sending generator to generator in cohomology, yields an isomorphism in homology through dimension 15. Similarly, the map

$$B(E_8 \times E_8) = BE_8 \times BE_8 \to K(Z,4) \times K(Z,4)$$

induces an isomorphism in homology through dimension 15. We use this isomorphism to compute the spin bordism of $B(E_8 \times E_8)$.

The motivation for this investigation was given by Witten (10), who examined a model for heterotic string theory for which an eleven-dimensional compact spin manifold M has 2 principal E_8-bundles $V_1 \oplus V_2 \to M$. Here the fundamental relations are that homotopy classes of maps of compact spin manifolds $g: M^n \to B(E_8 \times E_8)$ are in one-to-one correspondence with principal $E_8 \times E_8$-bundles $V_1 \oplus V_2 \to M^n$, but pairs (M^n, g) are elements of $\Omega_n^{\text{Spin}}(B(E_8 \times E_8))$. Corollary 7 shows that $\Omega_{11}^{\text{Spin}}(B(E_8 \times E_8)) = 0$, so that in fact an $E_8 \times E_8$-bundle over an 11-manifold must be trivial. This insures that the global space-time anomaly vanishes in the above mentioned model for string theory.

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