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# RELATIVE CHOW GROUPS

BY

STEVEN E. LANDSBURG

We shall propose a definition for the relative Chow groups of a scheme with respect to a closed subscheme and establish some basic properties. The definition generalizes to provide a definition of relative *higher* Chow groups as well.

In Section 1 we recall for reference the most important relationships between the classical (absolute) Chow groups and algebraic $K$-theory. In Section 2 we describe analogous relationships for the higher Chow groups introduced by Bloch in [B]. (These are canonically isomorphic to the higher "PreChow groups" of [L].) The results here are of independent interest. In Section 3 we introduce relative analogues of many important constructions. In Section 4 we define the relative Chow groups and relative higher Chow groups. We establish their basic properties and their relationship to $K$-theory, emphasizing the analogies between this material and that of Sections 1 and 2.

## 1. Absolute Chow groups and algebraic $K$-theory

We begin by recalling, for later reference, some of the main properties of the usual (absolute) Chow groups, particularly those that relate the Chow groups to algebraic $K$-theory.

**1.1.** Let $X$ be a regular scheme essentially of finite type over a field $k$. (Regularity can be relaxed in much of what follows.) We have the following invariants:

$Z^m(X)$, the group of codimension-$p$ algebraic cycles on $X$. That is, $Z^m(X)$ is free abelian on those reduced and irreducible closed subschemes of $X$ that have codimension $m$.

$Ch^m(X) = Z^m(X)/R^m(X)$, the $m$th Chow group of $X$. Here $R^m(X) \subset Z^m(X)$ is the subgroup consisting of cycles rationally equivalent to zero.

$K_m(X)$, the $m$th Quillen $K$-group of $X$.

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