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ON THE HURWITZ ZETA-FUNCTION

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1. Introduction

For $0 < \alpha \leq 1$, let $\zeta(s, \alpha)$ be the Hurwitz zeta-function defined by

$$\zeta(s, \alpha) = \sum_{n=0}^{\infty} (n + \alpha)^{-s} \quad \text{for } \operatorname{Re}(s) > 1$$

and its analytic extension,

$$\zeta_x(s, \alpha) = \zeta(s, \alpha) - \sum_{0 \leq n \leq x - \alpha} \frac{1}{(n + \alpha)^s}.$$

J.F. Koksma and C.G. Lekkerkerker [1] first studied the mean square value

$$f(s) = \int_0^1 |\zeta_1(s, \alpha)|^2 \, d\alpha,$$

and obtained the following results:

THEOREM A. *If* $1/2 < \sigma \leq 1$ *and* $|t| \geq 3$, *then*

(I) $$f\left(\frac{1}{2} + it\right) \leq 64 \ln|t|,$$

(II) $$\left| f(\sigma + it) - \frac{1}{2\sigma - 1} \right| \leq |t|^{1-2\sigma}\left(32 \ln|t| + \frac{1}{2\sigma - 1}\right).$$

Secondly, V.V. Rane [2] gave a general conclusion, namely:

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