ILLINOIS JOURNAL OF MATHEMATICS
Volume 36, Number 1, Spring 1992

ROOT NUMBERS OF JACOBI-SUM HECKE CHARACTERS

BY

DAVID E. ROHRLICH[1]

Let p be an odd prime and n a positive integer, and let K be the cyclotomic field of p^n-th roots of unity. Let a, b, and c be nonzero integers satisfying $a + b + c = 0$. We assume that none of the integers a, b, and c is divisible by p^n and that at most one of them is divisible by p. The unitary Jacobi-sum Hecke character χ associated to these data is defined as follows. Given a prime ideal $\mathfrak{l}$ of K, relatively prime to p, and an element x of the ring of integers of K, relatively prime to $\mathfrak{l}$, let $\left(\frac{x}{\mathfrak{l}}\right)_{p^n}$ denote the unique p^n-th root of unity such that

$$\left(\frac{x}{\mathfrak{l}}\right)_{p^n} \equiv x^{(N\mathfrak{l}-1)/p^n} (\mathrm{mod}\ \mathfrak{l}).$$

Put

$$J(\mathfrak{l}) = -\sum_x \left(\frac{x}{\mathfrak{l}}\right)_{p^n}^a \left(\frac{1-x}{\mathfrak{l}}\right)_{p^n}^b,$$

where x runs over representatives for the distinct residue classes modulo $\mathfrak{l}$, the classes of 0 and 1 being omitted. Now extend J by complete multiplicativity to the group $I(p)$ of fractional ideals of K relatively prime to p, and embed K into $\mathbf{C}$, so that J becomes a homomorphism from $I(p)$ to $\mathbf{C}^\times$. Then J is a Hecke character (Weil [8]). The associated unitary Hecke character is

$$\chi(\mathfrak{a}) = J(\mathfrak{a})(\mathrm{N}\mathfrak{a})^{-1/2},$$

where $\mathfrak{a}$ denotes an arbitrary element of $I(p)$.

In his original paper of 1952, Weil posed the problem of determining the conductor $\mathfrak{f}(\chi)$ of χ. While the case $n = 1$ was settled by Hasse [4] soon thereafter, the determination of $\mathfrak{f}(\chi)$ for arbitrary n was accomplished only recently, by Coleman and McCallum [1]. The present note gives an application of their result. At issue is the root number in the functional equation of

---

Received July 9, 1990
1980 Mathematics Subject Classification (1985 Revision). Primary 11R42.
[1]Research partially supported by a grant from the National Science Foundation.