ILLINOIS JOURNAL OF MATHEMATICS
Volume 36, Number 1, Spring 1992

THE SYMMETRIC GENUS OF THE HIGMAN-SIMS GROUP *HS* AND BOUNDS FOR CONWAY'S GROUPS Co_1, Co_2

BY

ANDREW J. WOLDAR[1]

Introduction

By a *surface* we shall always mean a closed connected compact orientable 2 manifold. For G a finite group, the *symmetric genus* $\sigma(G)$ *of* G is, by definition, the least integer g such that there exists a surface of genus g on which G acts in a conformal manner. It is well known that any such action of G on a surface S must be accompanied by an orientation-preserving action of G^0 on S, where G^0 is a subgroup of index at most 2 in G. In particular, if G is simple, its conformal action on S must be orientation-preserving. In this case we have $\sigma(G) = \sigma^0(G)$, where $\sigma^0(G)$ denotes the *strong symmetric genus of* G, defined to be the least integer g such that there is a surface of genus g on which G acts in an orientation-preserving manner.

In this paper we determine the symmetric genus of the Higman-Sims sporadic group *HS* and substantially improve existing bounds for the sporadic groups Co_1 and Co_2 of Conway. To do this we rely on the theory of triangular tesselations of the hyperbolic plane (e.g. see [2], [3], [4]), as well as a theorem of Tucker on partial presentations of groups which admit cellularly embedded Cayley graphs in surfaces of prescribed genus (see [7]). This reduces the problem to one of group generation, which can be handled in principal by computing relevant structure constants for the group, as well as for a variety of its subgroups, by means of character tables. (See [9] for additional details on all of the above remarks.) Throughout, we adopt the notation used in [1] and [8]. In particular, $\Delta_G(K_1, K_2, K_3)$ denotes the structure constant whose value is the cardinality of the set

$$\{(a, b) \colon a \in K_1, b \in K_2, ab = c\},$$

where c is a fixed element of the conjugate class K_3 of G. Also all conjugate classes are understood to be G-classes unless otherwise inferred.

Received May 7, 1990.
1980 Mathematics Subject Classification (1985 Revision). Primary 20D08

[1]This research was completed while the author was visiting the University of Delaware.