ILLINOIS JOURNAL OF MATHEMATICS
Volume 36, Number 3, Fall 1992

BANACH LATTICES WITH PROPERTY (H) AND WEAK HILBERT SPACES

BY

N.J. NIELSEN[1] AND N. TOMCZAK-JAEGERMANN

Introduction

The notions of weak type 2 and weak cotype 2 were introduced and studied by Milman and Pisier [10]. In [12] and [13] Pisier defined a weak Hilbert space to be a Banach space, which is both of weak type 2 and weak cotype 2 and developed an extensive theory of these spaces and weak properties in general. In [12] he defined the so-called property (H) for Banach spaces (which roughly says that for every normalized unconditional basic sequence (x_j) and for every integer $n, \|\sum_{j=1}^{n} x_j\|$ behaves like $\sqrt{n}$) and proved that weak Hilbert spaces have this property; it was left as an open problem whether property (H) is actually equivalent to the space in question being a weak Hilbert space.

One of the major problems of the theory is the scarcity of known examples; basically the only known weak Hilbert spaces are variations of the Tsirelson construction (see e.g., [2]) and this raises the question whether every weak Hilbert space has a basis.

In this paper we study the structure of unconditional sequences in Banach spaces with property (H) and we give strong estimates of the tail behaviour of such sequences. The estimates have the same order of magnitude as those obtained for the unit vector basis of the 2-convexified Tsirelson space and its dual. We then use these results to show that a Banach lattice has property (H) if and only if it is a weak Hilbert space, thus solving the above question of Pisier in the affirmative for Banach lattices. We also combine our estimates with the results of W.B. Johnson [4] to investigate the structure of subspaces of quotients of a Banach lattice with property (H). We show that every such space has a basis and give estimates for the uniformity function of the uniform approximation property. Again these estimates have the same order of magnitude as in the Tsirelson case.

Received October 8, 1990.

1991 Mathematics Subject Classification. Primary 46B20; Secondary 46B42.

[1]This work was initiated when the first named author visited Edmonton in the Fall of 1989. He wishes to thank the University of Alberta for the hospitality.