ILLINOIS JOURNAL OF MATHEMATICS
Volume 37, Number 1, Spring 1993

M-IDEALS OF COMPACT OPERATORS

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1. Introduction

If X is a Banach space and E is a subspace of X then E is called an M-ideal in X if X^* can be decomposed an l_1-sum $X^* = E^\perp \oplus_1 V$ for some closed subspace V of X^*. This notion was introduced by Alfsen and Effros [1].

For any Banach space X we denote by $\mathscr{L}(X)$ the algebra of all bounded operators on X and by $\mathscr{K}(X)$ the ideal of compact operators. The first non-trivial example of an M-ideal obtained (Dixmier [7]) is that $\mathscr{K}(l_2)$ is an M-ideal in $\mathscr{L}(l_2)$. Subsequently, there has been considerable work on studying spaces X for which $\mathscr{K}(X)$ is an M-ideal in $\mathscr{L}(X)$. It was shown in Lima [20] that $\mathscr{K}(l_p)$ is an M-ideal in $\mathscr{L}(l_p)$ when $1 < p < \infty$ and that similarly $\mathscr{K}(c_0)$ is an M-ideal in $\mathscr{L}(c_0)$. Cho and Johnson [6] (cf. [2], [31]) showed that if a subspace X of l_p for $1 < p < \infty$ has the compact approximation property then $\mathscr{K}(X)$ is an M-ideal in $\mathscr{L}(X)$. Conversely, it is known that any separable space X for which $\mathscr{K}(X)$ is an M-ideal in $\mathscr{L}(X)$ satisfies the conditions that X^* is separable and has the metric compact approximation property (Harmand-Lima [11]). Further X must be an M-ideal in X^{**} [21] and, if it has the approximation property, it has an unconditional finite-dimensional expansion of the identity [9], [19] from which it can be deduced that X can be $(1+\varepsilon)$—embedded in a space with a shrinking 1-unconditional basis [19].

The aim of this paper is to give a classification of those separable Banach spaces X such that $\mathscr{K}(X)$ is an M-ideal in $\mathscr{L}(X)$. Having achieved this classification, then some outstanding questions can be resolved.

Our main result is Theorem 2.4 which lists six equivalences. The most important conclusion (condition (5)) is that a separable Banach space X has the property that $\mathscr{K}(X)$ is an M-ideal in $\mathscr{L}(X)$ if and only if X satisfies a structural condition, which we call property (M), and there is a sequence of compact operators (K_n) such that $K_n \to I$ strongly, $K_n^* \to I$ strongly and $\lim_{n\to\infty}\|I - 2K_n\| = 1$. Property (M) is the requirement that if u, v satisfy

Received May 21, 1991.
1991 Mathematics Subject Classification. Primary 46B20; Secondary 46H05.
[1]This research supported by a grant from the National Science Foundation.

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