ILLINOIS JOURNAL OF MATHEMATICS
Volume 37, Number 1, Spring 1993

THE HELICAL TRANSFORM AND THE A.E. CONVERGENCE OF FOURIER SERIES

I. ASSANI[1]

Introduction

Let $(X, \mathscr{F}, \mu, \varphi)$ be a dynamical system, μ being an invertible measure preserving transformation on $(X, \mathscr{F}, \mu)$. The helical transform $H_\varepsilon f(x)$ of $f \in L^1(\mu)$ is the limit a.e. of

$$H_\varepsilon^n f(x) = \sum_{k=-n}^{n}{}' \frac{f(\varphi^k x) e^{2\pi i k\theta}}{k}$$

for each ε fixed. The existence of the limit is known from the results of A. Calderón [3] and M. Cotlar [5]. (The notation Σ_j' means that we delete in the sums the term corresponding to $j = 0$.)

DEFINITION 1. A measurable function f satisfies the Wiener-Wintner property (with respect to the dynamical system $(X, \mathscr{F}, \mu, \varphi)$ if there exists a single null set $N \in X$ off which the limit $H_\varepsilon f(x)$ exists for all $\varepsilon \in \mathbf{R}$.

DEFINITION 2. A measurable function f satisfies the strong Wiener-Wintner property (with respect to $(X, \mathscr{F}, \mu, \varphi)$ if off a single null set $\varepsilon \to H_\varepsilon f(x)$ is a continuous function.

By taking an invariant function (i.e., $f \circ \varphi = f$) the discontinuity property at 0 of

$$\varepsilon \to \sum_{k=-\infty}^{\infty}{}' \frac{e^{ik\varepsilon}}{k}$$

shows easily that not all functions satisfy the strong Wiener-Wintner property (S.W.W.). This property (S.W.W.) is more likely to hold when we are outside the Kronecker factor of φ (i.e., the closure of the linear span of the

Received May 10, 1991.
1991 Mathematics Subject Classification. Primary 28D05; Secondary 42A20, 42A50, 47A60.
[1]Research supported in part by a grant from the University Research Council and a grant from the National Science Foundation.