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# HARMONIC AND ISOMETRIC ROTATIONS AROUND A CURVE

L. NICOLODI[1] AND L. VANHECKE

## 1. Introduction

In this paper we initiate the study of *local rotations* around a smooth embedded curve $\sigma: [a, b] \to (M, g)$ in a Riemannian manifold $(M, g)$. These transformations are local diffeomorphisms which generalize in a natural way the rotations around a straight line in Euclidean space $E^n$. They are determined by means of a field of endomorphisms along the curve, (the so-called *rotation field*), which for each $m \in \sigma$ fix the tangent vectors of $\sigma$ and when restricted to the fibers of the normal bundle of $\sigma$ behave like linear isometries.

Reflections with respect to a curve provide a class of examples of such rotations. We refer to [2], [16], [17], [18], [19] for further details about their study.

When $\sigma$ reduces to a point we obtain the rotations around a point which in turn generalize the geodesic symmetries. Such rotations are used to define different classes of Riemannian manifolds, for example symmetric spaces, generalized symmetric spaces and $s$-manifolds (see [6], [12], [18]). Moreover, the properties of these rotations may be used to characterize some particular classes of Riemannian spaces. For example, it is proved in [3] that harmonic geodesic symmetries characterize locally symmetric spaces. This result has been extended in [15] to $s$-regular manifolds by using a special class of rotations around a point. Further, when $(M, g, J)$ is an almost Hermitian manifold, then the field $J$ provides a natural rotation field. The properties of the corresponding rotations may again be used to characterize special classes of almost Hermitian manifolds as is done in [14]. (See also [18] for the use of geodesic symmetries in Hermitian and symplectic geometry.)

In this paper we study similar problems for rotations around a curve $\sigma$. The main purpose is to study *harmonic* rotations. In Section 2 we give some preliminaries. Then, in Section 3, we define rotations and derive, in the analytic case, a set of necessary and sufficient conditions for *isometric* rotations. We use this in Section 4 where we consider harmonic rotations and

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