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DIMENSION, VOLUME, AND SPECTRUM OF A RIEMANNIAN MANIFOLD

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1. Introduction

We consider the Laplace operator Δ defined on a Riemannian manifold M. In local coordinates we have;

$$\Delta f = -\frac{1}{\sqrt{g}} \sum_{i,j} \frac{\partial}{\partial x^i}\left(g^{ij}\sqrt{g}\, \frac{\partial f}{\partial x^j} \right),$$

where g_{ij} is the metric tensor, $g^{ik}g_{kj} = \delta^i_j$, and $g = \det(g_{ij})$.

The spectrum of Δ on M is the set of eigenvalues for the eigenvalue problem given by the equation

$$\Delta\phi = \lambda\phi,$$

where in case M has boundary we require that $\phi = 0$ on ∂M. In the latter case the spectrum is called the Dirichlet spectrum. We will sometimes be interested in the case where the manifold with boundary of interest is (the closure of) a relatively compact connected domain D in a complete Riemannian manifold X. Since M (or $\overline{D}$) is assumed to be compact the spectrum is given by a sequence of nonnegative numbers;

$$0 \le \lambda_1 \le \lambda_2 \le \lambda_3 \le \cdots \uparrow \infty$$

counted with multiplicity.

Note. As a matter of convention we will refer to the (Dirichlet) spectrum of Δ on M simply as the spectrum of M. Also, all manifolds referred to in this paper will be assumed to be connected and all domains will be assumed to have smooth boundary.

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