ILLINOIS JOURNAL OF MATHEMATICS
Volume 37, Number 1, Spring 1993

CONVERSION FROM NONSTANDARD MATRIX ALGEBRAS TO STANDARD FACTORS OF TYPE II_1

TAKANORI HINOKUMA AND MASANAO OZAWA

1. Introduction

In the recent applications of nonstandard analysis the following method of research has been acknowledged to be useful: To find a construction of a standard object by taking a standard part of a nonstandard object which is a nonstandard extension of a standard object or a well-defined internal object. By this method we can construct a complicated mathematical structure from a much simpler structure in the nonstandard universe. Loeb's construction [7] of measure spaces from simpler internal measure spaces has been known as one of the most successful results along with this line. For Banach space theory Henson and Moore [5] have found a construction of Banach spaces, called nonstandard hulls, from internal Banach spaces, and succeeded in characterizing several deep properties of Banach spaces by simpler properties of the nonstandard hulls obtained from their nonstandard extensions. In this paper we will apply this method to the theory of operator algebras. We will give a construction of a factor of type II_1 from a much simpler internal matrix algebra, and investigate some properties of this factor by the methods of infinitesimal analysis and hyperfinite combinatrics.

To summarize our construction in advance, let ν be a nonstandard natural number in an $\aleph_1$-saturated nonstandard universe. Consider the internal algebra of $\nu \times \nu$ matrices over the internal complex numbers, and pay attention to two norms on this algebra. One is the operator norm and the other is the normalized Hilbert-Schmidt norm. Collect all matrices with finite operator norm, and identify two such matrices if the normalized Hilbert-Schmidt norm of their difference is infinitesimal. The resulting algebra, equipped with the quotient norm that comes from the operator norm, is our factor of type II_1. Section 2 covers the fact that this is really a von Neumann algebra and a factor of type II_1. Section 3 proves the nonseparability of its representations, and Section 4 proves that it is not approximately finite. The proofs of these two results simplifies considerably the corresponding proofs

Received October 10, 1989
1991 Mathematics Subject Classification. Primary 03H05; Secondary 46L35.