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# RNP AND CPCP IN LEBESGUE-BOCHNER FUNCTION SPACES

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In this paper we study the extremal structure of the unit ball of a Lebesgue Bochner function space. Throughout, $X$ will denote a Banach space, $B_X$ the unit ball, $S_X$ the unit sphere, $X^*$ the dual space of $X$, $(\Omega, \Sigma, \mu)$ a positive measure space, and $1 < p, q < \infty$ with $1/p + 1/q = 1$.

Let $K$ be a subset of $X$. A point $x$ in $K$ is a point of sequential continuity of $K$ if for every sequence $(x_n)$ in $K$, weak-$\lim_n x_n = x$ implies $\lim_n \|x_n - x\| = 0$. The point of sequential continuity is a generalization of the point of continuity. A space $X$ has the Kadec-Klee property if every point $x$ in $S_X$ is a point of sequential continuity of $B_X$.

It is well-known that if $(\Omega, \Sigma, \mu)$ is not purely atomic, then $L^p(\mu, X)$ with the Kadec-Klee property must be strictly convex. This result, due to M. Smith and B. Turett [ST], is one of the most surprising results in the theory of Lebesgue-Bochner function spaces. Our first main result (Theorem 2.2) asserts that if $(\Omega, \Sigma, \mu)$ is atom-free, then every point of sequential continuity of $B_{L^p}(\mu, X)$ must be an extreme point of $B_{L^p(\mu, X)}$. This gives a local version of the result of Smith and Turett.

Theorem 2.2 has several interesting consequences; for example, it implies that if $(\Omega, \Sigma, \mu)$ is not purely atomic then:

(i) The Radon-Nikodym Property (RNP) and the Convex Point of Continuity Property (CPCP) are equivalent for $L^p(\mu, X)$ and $L^p(\mu, X)^*$.

(ii) The super-RNP and the super-CPCP are equivalent for $L^p(\mu, X)$ and $L^p(\mu, X)^*$.

Recall that the RNP implies the PCP (Point of Continuity Property) which, in turn, implies the CPCP, and that RNP, PCP, and CPCP are distinct [BR], [GMS1]. It follows that if $X$ has the PCP but fails the RNP, and if $(\Omega, \Sigma, \mu)$ is not purely atomic, then $L^p(\mu, X)$ does not have the CPCP. Consequently, neither the PCP nor the CPCP can be "lifted" from $X$ to $L^p(\mu, X)$. We would like to mention (1) it is still an open problem whether the super-RNP and the super-CPCP are equivalent in general, (2) the RNP and the CPCP are equivalent for Banach spaces with the Krein-Milman Property [Sc], and

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