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LARGE DEVIATIONS FOR NONSTATIONARY ARRAYS AND SEQUENCES[1]

J. R. BAXTER, N. C. JAIN AND T. O. SEPPÄLÄINEN

1. Introduction

In the present paper we shall prove several results which apply to empirical distributions and empirical processes for nonstationary sequences of random variables. Our first result, Theorem 5.1, which deals with triangular arrays, will be derived from a theorem of Kifer [8], which gives a criterion for the large deviation principle to hold. Kifer's result is stated below in a general form as Theorem 3.5. A geometrical proof of Theorem 3.5 is given in [4]. Theorem 5.1 applies in particular to arrays of independent variables, as is pointed out in Corollary 5.4. Another criterion for the large deviation principle to hold is given in Theorem 6.5, which is a generalization of a result proved in [4]. Applications of Theorem 6.5 are given in Corollaries 7.1 and 8.1. Corollary 8.1 implies a large deviation result in the nonstationary hypermixing case, Theorem 9.13. In Section 10 it is shown that the results of this paper can be applied to the case of an independent sequence whose distributions are quasi-regular, in particular when the distributions are generated by a stationary random process.

A compactification argument will be used in the proofs of Theorems 5.1 and Corollaries 7.1 and 8.1. This step uses some simple compactification results from [4], which are stated in Proposition 4.1 and Proposition 4.9.

2. The LDP

Throughout this paper, for any σ-algebra $\mathscr{F}$, we let $\mathscr{M}(\mathscr{F})$, $\mathscr{M}_+(\mathscr{F})$, and $\mathscr{M}_1(\mathscr{F})$ denote the space of all bounded signed measures on $\mathscr{F}$, the space of all bounded nonnegative measures on $\mathscr{F}$, and the space of all probability measures on $\mathscr{F}$, respectively. By a scaling sequence $(r(n))$ we will mean a sequence of positive integers such that $r(n) \to \infty$ as $n \to \infty$. We will begin by stating the large deviation principle in a suitable form.

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