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ON SOME STRANGE SUMMATION FORMULAS

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1. Introduction

During the last two years the first named author used symbolic algebra programs and long hours of computer experiments to formulate several infinite series identities. Some of his conjectures were communicated to other mathematicians as informal letters, and were circulated among interested parties. In particular [10] and [11] contained several conjectures in the form of identities reminiscent of Ramanujan's work.

Some of the series relevant to this work are

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos\left(\sqrt{b^2+\pi^2 n^2}\right) = \frac{\pi^2}{4}\left(\frac{\sin b}{b} - \frac{\cos b}{3}\right), \tag{1.1}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos\left(\sqrt{n^2\pi^2 - 9}\right) = -\frac{\pi^2}{12e^3}, \tag{1.2}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+\frac{1}{2}} \frac{\sin\sqrt{b^2+\pi^2(n+1/2)^2}}{\sqrt{b^2+\pi^2(n+1/2)^2}} = \frac{\pi}{2}\frac{\sin b}{b}, \tag{1.3}$$

$$\sum_{-\infty}^{\infty} \frac{\cos\left[\sqrt{(n\pi+\phi+a/b)(n\pi+\phi+ab)}\right]}{(-1)^n(n\pi+\phi)^2} = \cot\phi\,\frac{\cos a}{\sin\phi} - \frac{(b+1/b)\sin a}{2\sin\phi}. \tag{1.4}$$

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