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ANTISTABLE CLASSES OF THIN SETS IN HARMONIC ANALYSIS

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Introduction

The motivation for this study is a property of the class $\mathscr{N}$ of all sets of absolute convergence (of a trigonometric series whose sum of coefficients is infinite): an increasing countable union of compact (or $\mathscr{K}_\sigma$) $\mathscr{N}$-sets is an $\mathscr{N}$-set (Host-Méla-Parreau [8]). Is it still true for any increasing countable union of $\mathscr{N}$-sets? This problem was posed by J. Arbault in [1]. It led me to study in general the operation of increasing countable union, and to a precise study of the class $\mathscr{N}$ and various related classes of thin sets. My Thèse de Doctorat [11], under the supervision of A. Louveau, contains some of the ideas developed in this paper.

Stability under finite union or countable union of classes of thin sets naturally introduced in harmonic analysis (e.g., sets of uniqueness [2] or Helson sets [18]) are classical problems (most of them are collected in the appendix of [17]). On the other hand, the stability of these classes under increasing countable union, to my knowledge, has never been studied. This paper can be considered as mixing harmonic analysis and descriptive set theory, in the same vein as the work done, in the study of sets of uniqueness, on σ-ideals and the operation of countable union [13]. But contrary to the operation of countable union, the operation of increasing countable union has no good descriptive properties [3]. In particular this operation is not idempotent, and ω_1 iterations are needed in general to obtain the closure of a class under this operation. The general study of the operation of increasing countable union and of related operations is done, from a combinatorial point of view, in [12].

The notion of a set of absolute convergence was introduced by P. Fatou in 1906 [7] and was successively studied by N. Lusin 1912 [19], V. V. Niemytzki 1926 [22], Marcinkiewicz 1938 [21], R. Salem 1941 [23], J. Arbault 1952 [1], J. E. Björk and R. Kaufman 1967 [18] and B. Host, J.-F. Méla and F. Parreau 1991 [8]. In the first section, we present the classical properties of the class $\mathscr{N}$

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