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# FUNDAMENTAL SOLUTIONS FOR POWERS OF THE HEISENBERG SUB-LAPLACIAN

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## 1. Introduction and statement of results

The *Heisenberg group* $H_n$ of dimension $2n + 1$ is given by

$$H_n := \mathbf{C}^n \times \mathbf{R} \tag{1.1}$$

with product

$$(z,t)(z',t') = \left(z + z', t + t' - \tfrac{1}{2}\operatorname{Im}(z \cdot \overline{z'})\right) \tag{1.2}$$

for $z, z' \in \mathbf{C}^n$, $t, t' \in \mathbf{R}$. Differentiation along the one-parameter subgroups

$$\{x_j(s) = (se_j, 0)\} \quad \text{and} \quad \left\{y_j(s) = \left(\sqrt{-1}\, se_j, 0\right)\right\},$$

where $\{e_j\}$ is the standard basis for $\mathbf{C}^n$, yields left invariant vector fields $X_j$ and $Y_j$ respectively. Letting $Z_j := X_j + \sqrt{-1}\, Y_j$ and $\overline{Z}_j := X_j - \sqrt{-1}\, Y_j$, one computes that

$$Z_j = 2\frac{\partial}{\partial \bar{z}_j} + \frac{\sqrt{-1}}{2} z_j \frac{\partial}{\partial t},$$

$$\overline{Z}_j = 2\frac{\partial}{\partial z_j} - \frac{\sqrt{-1}}{2} \bar{z}_j \frac{\partial}{\partial t}. \tag{1.3}$$

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