ILLINOIS JOURNAL OF MATHEMATICS
Volume 37, No. 3, Fall 1993

SOME PROPERTIES OF THE QUOTIENT SPACE $(L^1(T^d)\,/\,H^1(D^d))$

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1. Introduction

Let D be the unit disc of the complex plane and $\mathbf{T}$ the unit circle equipped with normalized Lebesgue measure. Let d be a positive integer. We denote by D^d and $\mathbf{T}^d$ respectively the products of d copies of D and $\mathbf{T}$. They are respectively the d-disc and d-torus. $\mathbf{T}^d$ is equipped with the product measure dm. Let $0 < p \leq \infty$. We denote by $H^p(D^d)$ the classical Hardy space in the polydisc D^d. If $p < \infty$, this is the space of the analytic functions f in D^d such that

$$\sup_{0 \leq r < 1} \int_{\mathbf{T}^d} |f(rz)|^p \, dm(z) < \infty,$$

where $r = (r_1, \ldots, r_d) \in [0, 1)^d$, $z = (z_1, \ldots, z_d) \in \mathbf{T}^d$ and $rz = (r_1 z_1, \ldots, r_d z_d) \in D^d$. If $p = \infty$, $H^\infty(D^d)$ is the space of the bounded analytic functions in D^d. Equipped with its natural norm or quasi-norm, $H^p(D^d)$ is a Banach space if $1 \leq p \leq \infty$ and a quasi-Banach space if $0 < p < 1$. It is well-known that every function in $H^p(D^d)$ admits a.e. radial limits on $\mathbf{T}^d$ and the function is uniquely determined by its boundary function on $\mathbf{T}^d$. Thus identifying functions in $H^p(D^d)$ with their boundary values, we may regard $H^p(D^d)$ as a closed subspace of $L^p(\mathbf{T}^d)$.

Let us recall that a Banach (or quasi-Banach) space X is of cotype 2 if there exists a constant C such that for all finite sequences $\{x_n\} \subset X$

$$\left(\sum \|x_n\|^2\right)^{1/2} \leq C \int \left\| \sum \varepsilon_n x_n \right\|,$$

where $\{\varepsilon_n\}$ is a Rademacher sequence. Recall also that a linear operator u: $X \to Y$ between two Banach spaces is called p-summing $(1 \leq p < \infty)$ if there

Received August 9, 1991
1991 Mathematics Subject Classification. Primary 46E15; Secondary 42B30.

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