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THE SYMMETRIC GENUS OF FINITE ABELIAN GROUPS[1]

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1. Introduction

A finite group G can be represented as a group of automorphisms of a compact Riemann surface [3]. In other words, there is a compact Riemann surface on which G acts and each non-identity element of G acts non-trivially on the surface. The *symmetric genus* $\sigma(G)$ is the minimum genus of any Riemann surface on which G acts faithfully. The *strong symmetric genus* $\sigma^\circ(G)$ is the minimum genus of any surface on which G acts faithfully and preserves the orientation. This terminology was introduced by Tucker [11].

Here we consider abelian groups acting on Riemann surfaces. Let A be a finite abelian group. The strong symmetric genus $\sigma^\circ(A)$ has been completely determined by Maclachlan [5]. Also the abelian groups of symmetric genus zero and one are well-known. We will calculate the symmetric genus $\sigma(A)$ in the case where $\sigma(A) \geq 2$ by using non-euclidean crystallographic groups (NEC groups). Our basic approach is to represent A as a quotient of an NEC group Γ by a surface group K, so that A acts on the surface U/K, where U is the open upper half-plane. We show that there is an action of A on a surface of least genus induced by an NEC group with a signature of one of three types. Groups of type I are Fuchsian groups and the corresponding action is orientation preserving. Groups of types II and III contain reflections. We denote by $\tau(A)$ the minimum genus of any action of A induced by an NEC group of type II. The number $\tau(A)$ depends on the relative sizes of the ranks of certain parts of A. The size of the largest elementary abelian 2-group direct summand of A determines whether $\sigma(A)$ is given by an action induced by a group of type I, II, or III. Our main result is the following.

THEOREM 5.7. *Let A be an abelian group of even order with canonical form $(Z_2)^a \times Z_{m_1} \times \cdots \times Z_{m_d}$ where $m_1 > 2$. If the symmetric genus $\sigma(A) \geq 2$,*

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