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A CHARACTERIZATION OF CYLINDERLIKE SURFACES

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Introduction

In this paper we wish to characterize the rings whose spectra are cylinderlike surfaces. We define these surfaces as follows.

DEFINITION 1. A cylinderlike surface is an affine, rational, nonsingular surface, $T = \operatorname{Spec} A$, over an algebraically closed field k of characteristic zero having the properties that $A^* = k^*$, $\operatorname{Pic} T$ is torsion, and T contains a nonempty subset U which is isomorphic to $\mathbf{A}^1 \times C$ where C is a rational curve. We will call a subset such as U a cylindrical open set.

Interest in these surfaces arose from a result of Miyanishi [M, Theorem 0], which says that if T is a cylinderlike surface and $\operatorname{Pic} T = 0$, then $T \simeq \mathbf{A}^2$. In certain applications of this result it is clear that $\operatorname{Pic} T$ is torsion and the main difficulty is to show that $\operatorname{Pic} T = 0$. This led to the consideration of cylinderlike surfaces. In general, Miyanishi's theorem fails for these surfaces. In fact, for any finite Abelian group G, there is a cylinderlike surface T with $\operatorname{Pic} T = G$. Theorem 4.1 of this paper gives an algebraic construction of such a surface. We are interested in the number of ways in which T can be chosen for any particular G.

In an attempt to answer this question, we have been considering a certain type of fibration on a cylinderlike surface.

DEFINITION 2. Suppose $T = \operatorname{Spec} A$ is a cylinderlike surface. We say that a morphism $k[x] \hookrightarrow A$ gives a cylindrical fibration of T if for each $\alpha \in k$ the fibre defined by $x - \alpha$ is irreducible and if the complement of a finite set of these fibres is a cylindrical open set.

One previously obtained result, which is due to Richard Swan and which will be proved as Proposition 1.1 of this paper, says that if $T = \operatorname{Spec} A$ is a cylinderlike surface, then there are $\alpha_i \in k$, $n_i \in \mathbf{Z}^+$, and prime ideals P_i of

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