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THE STONE-WEIERSTRASS PROPERTY IN QUOTIENT ALGEBRAS, AND SETS OF SPECTRAL RESOLUTION

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1. Introduction

In 1960, Katznelson and Rudin, motivated by the Schwartz counterexample and Malliavin's theorem, extended the notion of the Stone-Weierstrass property to semi-simple commutative Banach algebras [9]. Since the Schwartz counterexample to the spectral synthesis can easily be modified to an example of a strongly separating self-adjoint subalgebra which is not dense in $A(R^3)$, and since Malliavin showed, in 1959, that $A(G)$ is an algebra of synthesis if and only if the LCA group G is discrete [12], [13], [14], Katznelson and Rudin were interested in investigating the Stone-Weierstrass property in $A(G)$. They concluded that $A(G)$ is a Stone-Weierstrass algebra if and only if G is totally disconnected [9], [16, Section 9.3]. Since every discrete group is totally disconnected, we can observe that every algebra $A(G)$ of synthesis is a Stone-Weierstrass algebra, or equivalently, if $A(G)$ does not have the Stone-Weierstrass property, then G contains a non-S-set. The converse is false.

In this paper we investigate the Stone-Weierstrass property in quotient algebras $A(E)$, where E is a closed subset of an LCA group. We define two classes of sets, Stone-Weierstrass sets and idempotent sets, and observe the relation between these sets and sets of spectral resolution. In this case the situations are very different from the case of an LCA group. First, the assumption "E is a set of spectral resolution" does not imply "E is discrete." A perfect Kronecker set in T (cf. [16, p. 99]) is a counterexample. If E is discrete, however, E is a set of spectral resolution (cf. [16, p. 159]). Second, even if $A(E)$ is a Stone-Weierstrass algebra, E may not be totally disconnected. Helson curves in T^n, $n \geq 2$, constructed by Kahane and McGehee serve as counterexamples [7], [15]. Third, even if E is totally disconnected, $A(E)$ may not be a Stone-Weierstrass algebra. Katznelson and Rudin constructed a counterexample [9, p. 257].

Our main results are as follows. First, every closed subset of an idempotent set and a Stone-Weierstrass set is an idempotent set and a Stone-Weierstrass

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