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ALMOST EVERYWHERE CONVERGENCE OF CONVOLUTION POWERS IN $L^1(X)$

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Introduction

This paper is concerned with the behavior of weighted averages induced by a probability measure on the integers. Let (X, β, m) be a probability space and τ: $X \mapsto X$ an invertible measure preserving point transformation. A probability measure μ on $\mathbf{Z}$, the integers, gives rise to the weighted average

$$\mu f(x) = \sum_{k=-\infty}^{\infty} \mu(k) f(\tau^k x).$$

The powers of the operator μf are defined by the convolution powers of the measure μ,

$$\mu^n f(x) = \sum_{k=-\infty}^{+\infty} \mu^n(k) f(\tau^k x)$$

where, on the right hand side, $\mu^n(k)$ denotes the nth convolution power of μ evaluated at k. Note that since

$$\left(\int |\mu^n f(x)|^p \, dm(x)\right)^{1/p} \leq \sum_{k \in \mathbf{Z}} \mu(k) \left(\int |f(\tau^k x)|^p \, dm(x)\right)^{1/p} = \|f\|_p,$$

these operators are well defined a.e. and are positive contractions in all $L^p(X)$, $1 \leq p \leq \infty$. Bellow-Jones-Rosenblatt [2], [3], [5] studied these types of averaging operators as well as more general types of weighted averages. They proved these operators converge in norm whenever the support of μ is not contained in a coset of a proper subgroup of $\mathbf{Z}$. In addition they proved [3] that if the measure is centered and has finite second moment then there is convergence almost everywhere in $L^p(X)$ for all $p > 1$. Their method is based on Fourier techniques that could not be extended to L^1. V. I. Oseledec [14] proved convergence almost everywhere in L^1 for symmetric

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