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ON A THEOREM OF BURKHOLDER

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Let $\{r_k(t)\}_{k=0}^{\infty}$ be Rademacher functions defined as

$$r_0(t) = \begin{cases} 1 & \text{if } 0 \le t < \frac{1}{2} \\ -1 & \text{if } \frac{1}{2} \le t < 1 \end{cases}$$

$$r_0(t+1) = r_0(t);$$

$$r_k(t) = r_0(2^k t).$$

E.M. Stein, in his important paper [3], applied the following result: Let E be any measurable subset of $[0,1]$ and $|E| > 0$, then there is an integer N and a constant A both depending only on E such that if $c, c_1, c_2, \ldots$ are complex numbers and the series $\sum_{k=0}^{\infty} c_k r_k(t)$ converges almost everywhere, then

$$A\left(\sum_{k=N}^{\infty} |c_k|^2\right)^{1/2} \le \operatorname{esssup}\left\{\left|c + \sum_{k=0}^{\infty} c_k r_k(t)\right| : t \in E\right\}. \tag{1}$$

Rademacher functions are a sequence of independent random variables. D.L. Burkholder, in [1], extended (1) to other sequences of independent random variables satisfying certain conditions. In fact, Burkholder's result when specialized to Rademacher functions, is considerably stronger than (1). It is proved that there exist positive constants α and β so that for every set E, $|E| > 0$, there exists $N = N(E)$ so that

$$\left|\left\{t \in E: \beta\left(\sum_{k=N}^{\infty} |c_k|^2\right)^{1/2} \le \left|c + \sum_{k=0}^{\infty} c_k r_k(t)\right|\right\}\right| \ge \alpha|E|.$$

Using recently obtained norm inequalities for lacunary Walsh series [2] we extend Burkholder's theorem to q-lacunary Walsh series with $q > 1$. Since lacunary Walsh functions do not form an independent system of random variables, this case is not covered by Burkholder's theorem. Our proof is also

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