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FOLIATIONS INVARIANT UNDER THE MEAN CURVATURE FLOW

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Introduction

Let $\mathcal{F}$ be a foliation of a Riemannian manifold (M, g) equipped with the Levi-Civita connection ∇. The tangent bundle of M splits into the orthogonal sum $T\mathcal{F} \oplus T^{\perp}\mathcal{F}$ of the bundle tangent to F and its orthogonal complement, so any vector v decomposes into the sum $v^{\top} + v^{\perp}$ of vectors respectively tangent and perpendicular to $\mathcal{F}$. For any point $x \in M$, $H(x)$ denotes the mean curvature at x of the leaf L of $\mathcal{F}$ which passes through x. H is defined as the trace of the second fundamental form B of $\mathcal{F}$:

$$B(X,Y) = (\nabla_X Y)^{\perp} \tag{1}$$

for all vector fields X and Y tangent to $\mathcal{F}$ and

$$H = \sum_{i=1}^{p} B(X_i, X_i), \tag{2}$$

where $p = \dim \mathcal{F}$ and $X_1, \dots, X_p$ is a (local) orthonormal frame of vector fields tangent to $\mathcal{F}$. (For suitable background in Riemannian geometry we refer to [K], for the notions and results of the theory of foliations to [CN], [HH] and [T].)

In this paper, we are interested in those foliations $\mathcal{F}$ which are invariant under the local flows generated by the vector field H. Such foliations are said to be *mean curvature invariant*, or MCI for short. The infinitesimal condition sufficient and necessary for $\mathcal{F}$ to be MCI is that

$$\langle [H, X], N \rangle = 0 \tag{3}$$

for all vector fields X tangent to $\mathcal{F}$ and N orthogonal to $\mathcal{F}$. In other words, $\mathcal{F}$ is MCI iff H is parallel w.r.t. the (partial) Bott connection in $TM/T\mathcal{F} \cong T^{\perp}\mathcal{F}$, i.e. iff the Lie derivation $\mathcal{L}_H$ maps the module $\mathcal{X}(\mathcal{F})$ of vector fields

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