

## RELATION OF THE VAN EST SPECTRAL SEQUENCE TO $K$ -THEORY AND CYCLIC HOMOLOGY

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### 1. Introduction

In this paper we study how the smooth cohomology of the infinite general linear group  $GLA$  for a Banach algebra  $A$  relates to cyclic cohomology, Lie algebra cohomology and Dennis trace. Our main result is as follows.

**THEOREM A.** *The following diagram is commutative:*

$$\begin{array}{ccc}
 HH_c^*A & \xrightarrow{B} & HC_c^{*-1}A \\
 D_{sm} \downarrow & & \mathcal{A} \downarrow \\
 H_{sm}^*GLA & \xrightarrow{\lambda} & H_{Lie}^*gl\alpha
 \end{array}$$

Here  $B$  is the boundary map in Connes' long exact sequence relating continuous cyclic cohomology to continuous Hochschild cohomology [C].  $\mathcal{A}$  denotes the dual of the alternation operation that induces an isomorphism between the primitive elements in the Lie algebra homology of  $gl\alpha = MA$  and the cyclic homology of  $A$  [LQ] [T].  $\lambda$  is the classical map from the smooth cohomology of a group to its Lie algebra cohomology, which can be identified with one of the edge homomorphisms in the van Est spectral sequence. The definition of  $D_{sm}$  will rest on the observation that the dual of the Dennis trace map factors through the smooth group cohomology of  $GLA$ .

We incorporate the above diagram into a bigger commutative diagram to show its relation with the van Est spectral sequence and the various other well-known cohomology groups associated with a topological group.

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