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$\mathcal{M}$-SUBSPACES OF X_λ

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1. Introduction

Throughout this paper, n is a fixed positive integer, p, q, s, t nonnegative integers and α, λ are complex numbers related by $\lambda = -4n^2\alpha(1-\alpha)$.

1.1. Invariant Laplacian $\tilde{\Delta}$. B denotes the open unit ball of $\mathbf{C}^n$ with its boundary ∂B and Aut(B) the group of all bijective holomorphic maps of B onto itself. The invariant Laplacian $\tilde{\Delta}$ is defined by

$$(\tilde{\Delta} f)(z) = 4(1-|z|^2) \sum_{j,k=1}^{n} (\delta_{jk} - z_j \bar{z}_k) \frac{\partial^2 f}{\partial z_j \partial \bar{z}_k}(z), \quad f \in C^2(B),$$

where δ_{jk} is the Kronecker's symbol. It is invariant under Aut(B) in the sense that

$$\tilde{\Delta}(f \circ \varphi) = (\tilde{\Delta} f) \circ \varphi, \quad \varphi \in \operatorname{Aut}(B).$$

1.2. $\mathcal{H}_s$ and $H(p,q)$. $\mathcal{H}_s$ denotes the space of all homogeneous polynomials on $\mathbf{C}^n$ of degree s that satisfy $\Delta f = 0$ where

$$\Delta = 4 \sum_{j=1}^{n} \frac{\partial^2}{\partial z_j \partial \bar{z}_j}$$

is the ordinary Laplacian. The term "homogeneous" refers here to real scalars: $f(tz) = t^s f(z)$, $t > 0$.

Being harmonic, each $f \in \mathcal{H}_s$ is uniquely determined by its restriction on ∂B. These restrictions are so-called spherical harmonics. We shall freely identify $\mathcal{H}_s$ with its restrictions on ∂B.

$H(p,q)$ denotes the vector space of all harmonic homogeneous polynomials on $\mathbf{C}^n$ that have total degree p in the variables $z_1, \dots, z_n$ and total degree q in the variables $\bar{z}_1, \dots, \bar{z}_n$. Some of the basic properties of $H(p,q)$

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