ILLINOIS JOURNAL OF MATHEMATICS
Volume 38, Number 1, Spring 1994

ON THE INTEGRATION OF VECTOR-VALUED FUNCTIONS

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Introduction

A large number of different methods of integration of Banach-space-valued functions have been introduced, based on the various possible constructions of the Lebesgue integral. They commonly run fairly closely together when the range space is separable (or has w^*-separable dual) and diverge more or less sharply for general range spaces. The McShane integral as described by [7] is derived from the 'gauge-limit' integral of [11]. Here we answer some questions left open in [7] concerning the relationship between the McShane and Pettis integrals. Our original objectives were simply to confirm that McShane integrable functions are Pettis integrable (2C) and to find a Pettis integrable function which is not McShane integrable. Seeking interesting examples of such functions we were led to investigate the connections between the McShane and Talagrand integrals (2L, 2M, 3A, 3C). As far as we know, the 'Talagrand integral' of 1Ab below is explicitly described here for the first time, although all the significant facts we use are given in [13] and [14].

Perhaps we should make some remarks on the context of our results. The ordinary functional analyst is naturally impatient with the multiplicity of definitions of 'integral' which have been proposed for vector-valued functions, and would much prefer to have a single canonical one for general use. In our view the only integral with any claim to such pre-eminence is the Bochner integral (1Ac). But elementary examples (3D below is a classic) show that the Bochner integral is highly restrictive, in that it integrates few functions; and quite simple problems lead us to demand extensions. In this paper we work with three such extensions. We hope that our positive results (e.g., McShane integrable functions are Pettis integrable) will make the jungle seem a little less impenetrable, while our negative results (e.g., the domains of the McShane and Talagrand integrals are incomparable) will at least clarify the irreducible difficulties of the subject. In passing, we prove a weak convergence theorem for the McShane integral (2I-2J), showing that

Received February 13, 1992.
1991 Mathematics Subject Classification. Primary 28B05; Secondary 46G10.