ILLINOIS JOURNAL OF MATHEMATICS
Volume 38, Number 1, Spring 1994

SOME REMARKS ON EXTENSION THEOREMS FOR WEIGHTED SOBOLEV SPACES

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1. Introduction

Let $\mathscr{D}$ be an open set in $\mathbf{R}^n$. If α is a multi-index, $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbf{Z}_+^n$, we will denote $\Sigma_{j=1}^n \alpha_j$ by $|\alpha|$ and let

$$D^\alpha = \left(\frac{\partial}{\partial x_1}\right)^{\alpha_1} \cdots \left(\frac{\partial}{\partial x_n}\right)^{\alpha_n}.$$

A locally integrable function f on $\mathscr{D}$ has a weak derivative of order α if there is a locally integrable function (denoted by $D^\alpha f$) such that

$$\int_{\mathscr{D}} f(D^\alpha \varphi)\, dx = (-1)^{|\alpha|} \int_{\mathscr{D}} (D^\alpha f) \varphi\, dx$$

for all C^∞ functions φ with compact support in $\mathscr{D}$ (we will write $\varphi \in C_0^\infty(\mathscr{D})$).

By a weight w, we mean a nonnegative locally integrable function on $\mathbf{R}^n$. By abusing notation, we will also write w for the measure induced by w. Sometimes we write dw to denote $w\,dx$. We always assume w is doubling, by which we mean $w(2Q) \le Cw(Q)$ for every cube Q, where $2Q$ denotes the cube with the same center as Q and twice its edgelength. Let μ be another weight. By $w/\mu \in A_p(\mu)$, we mean

$$\frac{1}{\mu(Q)} \left(\int_Q \frac{w}{\mu}\, d\mu\right)^{1/p} \left(\int_Q \left(\frac{w}{\mu}\right)^{-1/(p-1)} d\mu\right)^{(p-1)/p} \le C \text{ when } 1 < p < \infty, \text{ and}$$

$$\frac{\mu(x)}{\mu(Q)} \le C \frac{w(x)}{w(Q)} \text{ a.e. when } p = 1,$$

for all cubes Q in $\mathbf{R}^n$. If Q is a cube, let $l(Q)$ be the edgelength of Q. For $1 \le p \le \infty$, $k \in \mathbf{N}$, and any weight w, $L_{w,k}^p(\mathscr{D})$ and $E_{w,k}^p(\mathscr{D})$ are the spaces

Received Feb. 11, 1992.
1991 Mathematics Subject Classification. Primary 46E35.