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FEJER THEOREMS ON COMPACT SOLVMANIFOLDS

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Section 1. Introduction

In the theory of Fourier series, it is well known that if $f \in L^2(T)$ is continuous, where T is the unit circle), then the Fourier series

$$g(x) = \sum_{n \in \mathbf{Z}} \hat{f}(n) e^{2\pi i n x}$$

need not converge uniformly or even pointwise to f. However, the Fejer theorem asserts that there exists a set of constants $\{a_{n,k}\}_{n,k=1}^{\infty}$, such that for each fixed n only finitely many k differ from 0, and so that if we define

$$\sigma_n(x) = \sum_{k \in \mathbf{Z}} a_{n,k} \hat{f}(k) e^{2\pi i k x},$$

then $\sigma_n \to f$ uniformly on T.

We note that the map $f \mapsto \hat{f}(n)e^{2\pi i n x}$ is an orthogonal projection onto a subspace of $L^2(T)$ which is translation-invariant; if Λ denotes the quasi-regular representation of $\mathbf{R}$ in $L^2(T)$, then Λ restricted to the subspace $\{\mathbf{C}e^{2\pi i n x}\}$ is equivalent to an irreducible representation of $\mathbf{R}$.

Similarly, if S is a solvable Lie group with cocompact discrete subgroup Γ, the right quasiregular representation decomposes $L^2(S/\Gamma)$ into a countable direct sum of orthogonal irreducible subspaces. Those irreducible representations of S which appear in the decomposition may appear with multiplicity, always finite. Although the decomposition of $L^2(S/\Gamma)$ isn't unique, the direct sum of all irreducible π-spaces is independent of the decomposition; we call it the primary summand of π. We order the primary summands $\{H_n\}$ and let P_n denote orthogonal projection onto the nth primary summand.

In this paper we address the question of whether Fejer theorems exist for the three-dimensional compact solvmanifolds which are quotients of the

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