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ON THE MEAN SQUARE VALUE OF THE HURWITZ ZETA-FUNCTION

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1. Introduction

For a complex number $s = \sigma + it$ and a real number $0 < \alpha < 1$ let $\xi(s, \alpha)$ be the Hurwitz zeta-function defined by

$$\xi(s, \alpha) = \sum_{n=0}^{\infty} \frac{1}{(n+\alpha)^s}$$

for $\mathrm{Re}(s) > 1$, and its analytic continuation for $\mathrm{Re}(s) \leq 1$, and let $\xi_1(s, \alpha) = \xi(s, \alpha) - \alpha^{-s}$.

The main purpose of this paper is to study the asymptotic properties of the mean square value

$$\int_0^1 \xi_1(\sigma_1 + it, \alpha)\xi_1(\sigma_2 - it, \alpha)\, d\alpha \tag{1}$$

where $0 < \sigma_1, \sigma_2 < 1$ and t is an arbitrary real number.

V. V. Rane [1] proved that

$$\int_0^1 |\zeta_1(\tfrac{1}{2} + it, \alpha)|^2\, d\alpha = \ln t + O(1)$$

holds uniformly in $t > 2$.

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