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HOMOLOGICAL PROPERTIES OF STRATIFIED SPACES

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In [4], Goresky and MacPherson introduced intersection homology in order to extend Poincaré Duality to some singular spaces. They also introduced the intersection cohomology from a differential point of view by means of intersection differential forms [3]. Using the sheaf axiomatic construction of [5], it is shown in [3] that the intersection homology is dual to the intersection cohomology. Moreover, a subcomplex of intersection differential forms is exhibited in [2] for which the usual integration $\int$ of differential forms on simplices realizes the above duality (deRham Theorem). The context of these works is the category of Thom-Mather stratified spaces.

Later, MacPherson introduced a more general notion of intersection homology, by enlarging the notion of perversity [8]. The aim of this work is to extend the previous deRham Theorem to this new context; we also give a weaker presentation of intersection differential forms. The description of the "allowability condition" for intersection differential forms uses the tubular neighborhoods of the strata, it is a *germ* condition. It seems more natural to give a presentation of intersection differential forms whose "allowability" is measured more directly on the strata, as for the intersection homology.

Since the differential forms cannot be defined on the singular part of A, the version we propose here uses a *blow up* $\pi\colon \tilde{A} \to A$ of the stratified space (essentially the resolution of singularities of Verona [14]). The allowability of the differential forms is measured on the desingularization $\pi^{-1}(S)$ of the strata S of A. This gives rise to weak intersection differential forms. We show that the complex of these differential forms calculates the intersection homology of A. The proof is direct; that is, we show that the usual integration $\int$ of differential forms on simplices realizes the isomorphism. We finish the work by giving a direct proof of the fact that the Poincaré Duality for intersection cohomology ($IH^{\bar{p}}_{*}(A) \cong IH^{\bar{q}}_{n-*}(A)$) can be realized by the integration $\int$ of the usual wedge product of differential forms (see also [3] for classical perversities).

In Section 1 we recall the notion of a stratified space A and we introduce the *blow up* of A, the unfolding (in fact, the resolution of singularities of Verona without faces). Remark that in some cases the unfolding of A

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