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INTERMITTENT OSCILLATION AND TANGENTIAL GROWTH OF FUNCTIONS WITH RESPECT TO NAGEL-STEIN REGIONS ON A HALF-SPACE

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1. Introduction

Let n be a positive integer and denote the upper half-space $\mathbf{R}^n \times (0, \infty)$ in $\mathbf{R}^{n+1}$ by $\mathbf{R}_+^{n+1}$. The boundary $\partial\mathbf{R}_+^{n+1}$ of $\mathbf{R}_+^{n+1}$ will be identified, in the usual way, with $\mathbf{R}^n$. In the discussion below, we shall transpose (without explicit mention) results originally stated relative to the unit disk and its circumference in the complex plane, to the upper half-space $\mathbf{R}_+^{n+1}$ and its boundary $\mathbf{R}^n$ in $\mathbf{R}^{n+1}$.

In 1968, Å. Samuelsson [Sa] studied, for $n = 1$, the generalized derivatives of positive, Borel measures μ defined on $\mathbf{R}^n$ in relation to growth along the normal $N_x = \{(x, t)\colon 0 < t < \infty\}$ of the positive harmonic functions $\mathcal{K}\mu$ associated by means of the Poisson integral formula,

$$(1) \qquad \mathcal{K}\mu(x, t) = \int_{\mathbf{R}^n} K(x, t, z)\, d\mu(z), \qquad (x, t) \in \mathbf{R}_+^{n+1}.$$

Here, $K(x, t, z)$, for $x, z \in \mathbf{R}^n$ and $t \in (0, \infty)$, denotes the Poisson kernel for the upper half-space $\mathbf{R}_+^{n+1}$, and the measures μ satisfy the usual integral condition required for the convergence of the Poisson integral. Among other things, Samuelsson considered generalized upper symmetric derivatives of μ with respect to functions such as $\omega(t) = t^\beta$, $0 < \beta < 1$, at a point $x \in \mathbf{R}$, defined by

$$\overline{D}_\omega \mu(x) = \limsup \frac{|\mu(I)|}{\omega(|I|)},$$

where the intervals I are centered at x and the limit superior is taken as their lengths $|I|$ converge to 0. When $\overline{D}_\omega \mu(x)$ is positive, there is a sequence of intervals $\{I_j\}$ centered at x such that $|I_j| \to 0$ and $\mu(I_j)$ is (at least) of the order of $\omega(|I_j|)$. One may describe this roughly by saying that μ has

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